

Novel Transversity Properties in Hard Processes

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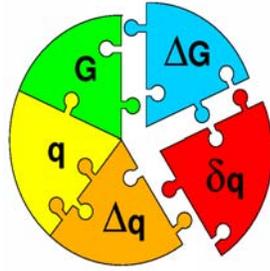
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- Historical Remarks: **Transversity**
- **Transversity in SSA and Azimuthal Asymmetries**
 - ★ T-even and T-odd contributions
- **Rescattering Mechanism in T-odd Structure and Fragmentation Functions**
 - ★ Novel Transversity Properties of the Nucleon
 - ★ Double T -odd $\cos 2\phi$ asymmetry
- **Model Estimates of Asymmetries**
- **Conclusions**

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Introductory Remarks: Transversity



- Transversity " δq " as combinations of helicity states

$$|\perp/\top\rangle = \frac{1}{\sqrt{2}} (|+\rangle \pm |-\rangle)$$

variable introduced by Goldstein & Moravcsik, Ann. Phys. 1976
reveal underlying simplicity spin-dependent nucleon-nucleon scattering amps, $f_{a,b;c,d}(s, t)$

- Connection with spin structure of nucleon revealed through the quark distribution

$$h_1^a(x) = \delta q^a(x) \text{ Artru \& Mekhfi, ZPC:1990}$$

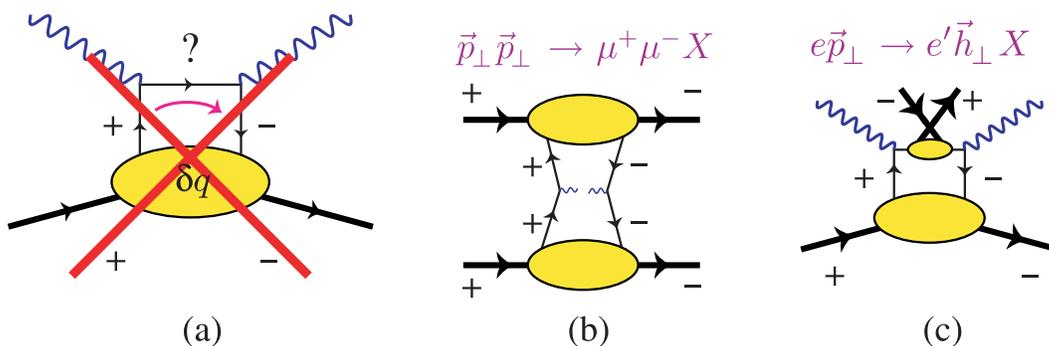
and first moment, tensor charge Jaffe & Ji, PRL:1991

$$\int_0^1 (\delta q^a(x) - \delta \bar{q}^a(x)) dx = \delta q^a$$

- ★ Transversity as fundamental to our understanding of spin structure nucleon as helicity $\Delta q^a(x)$: Dedicated programs BNL, HERMES, COMPASS & JLAB
- ★ Theoretically among other things: Soffer's inequality (Soffer, PRL:1995 & Goldstein Jaffe Ji, PRD:1995) suggestive bounds among leading twist distributions

$$|2\delta q^a| \leq q^a + \Delta q^a$$

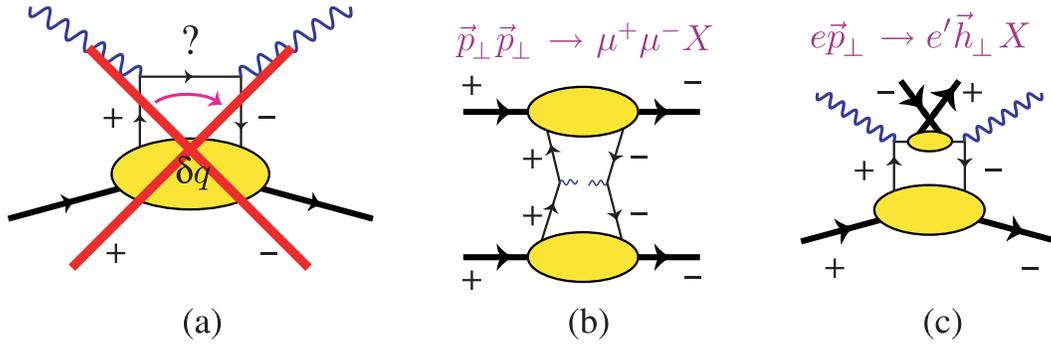
- ★ NLO analysis performed Vogelsang, PRD: 1998; indicates this bound is respected
- Decouples leading twist DIS: Helicity of struck quark must flip to probe transversity of nucleon **chiral-odd**



- \Rightarrow require a chiral-odd partner \Rightarrow and one or more hadrons involved in process to probe transversity ...
Jaffe: 2000 ...

Transversity and Hard scattering

- Drell-Yan $p_{\perp} + p_{\perp} \Rightarrow l^{+} l^{-} + X$ (2 in the initial state)
SIDIS $l + p_{\perp} \Rightarrow l' h + X$ (1 in the initial 1 in the final)



- ★ DY: Ralston and Soper NPB:1979 encountered in transverse spin asymmetries

$$A_{TT}^{DY} = \frac{2 \sin^2 \theta \cos 2\phi \sum_a e_a^2 h_1^a(x) \bar{h}_1^a(x)}{1 + \cos^2 \theta \sum_a e_a^2 f_1^a(x) \bar{f}_1^a(x)}$$

- ★ SIDIS: Jaffe and Ji PRL:1993 encountered at twist three level
Estimate of this effect, Gamberg, Hwang, Oganessyan PLB:2004

$$A_{LT} = \frac{\lambda_e |\mathbf{S}_T| \sqrt{1-y} \frac{4}{Q} \cos \phi_S \left[M x g_T(x) D_1(z) + M_h h_1(x) \frac{E(z)}{z} \right]}{\frac{[1+(1-y)^2]}{y} f_1(x) D_1(z)}$$

Experiments

- Large SSA are observed in $P P^\uparrow \rightarrow \pi X$
E704 Collaboration (1991), *AGS* (1999); *STAR* (2002)
- In
 - $\ell \vec{p} \rightarrow \ell' \pi X$ *HERMES* (1999, 2003), *CLAS* (2002)
 - $\vec{\ell} \vec{p} \rightarrow \ell' \pi X$ *CLAS* (2002)
 - *COMPASS* ...

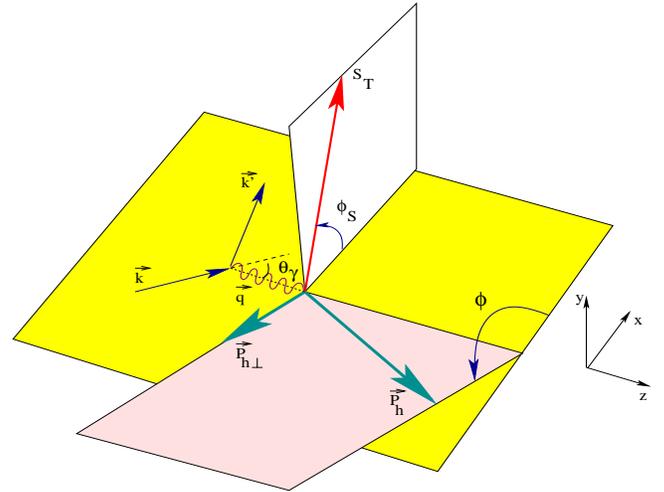
SIDIS and Transversity: Leading Twist

- SIDIS: Collins NPB:1993, Kotzinian NPB:1995, Mulders & Tangermann PLB:1995 transversity can be measured via azimuthal asymmetry in the fragmenting hadron's momentum (so called Collins effect:)

$P_{h\perp}$ - hadron transverse momentum

ϕ , azimuthal angle between (kq) and $(P_h q)$ planes

ϕ_S , azimuthal angle of the target spin vector

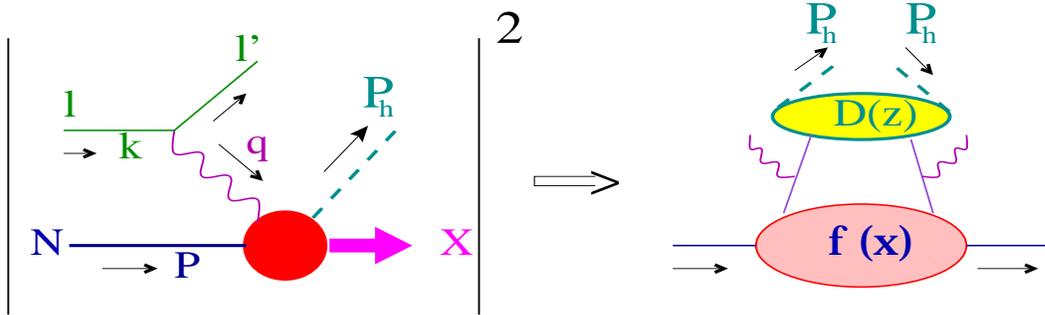


$$A_{UT}^{\sin(\phi+\phi_S)} \propto \frac{\sum_q e_q^2 h_1^q(x, Q^2) \cdot H_1^{\perp(1)q}(z, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) \cdot D_1^q(z, Q^2)}$$

Non-zero azimuthal asymmetries

- Origins

- ★ In the naive parton model all azimuthal asymmetries are zero at leading order
- ★ Both intrinsic k_T dependence and higher order in α_s in PQCD generate azimuthal asymmetries



$\cos 2\phi$ may arise from different mechanisms containing ordinary T-even structures *e.g.*

- ★ H. Georgi, H.D. Politzer, PRL:1978: pQCD first order α_s
- ★ R.N. Cahn, PLB: 1978; PRD: 1989; Oganessyan *et al.* , ZPC: 1998
Intrinsic k_{\perp}
- ★ E Berger, ZPC: 1980 289. Nonperturbative bound-state effects in pion wave function-higher twist

SIDIS and Hadronic Tensor

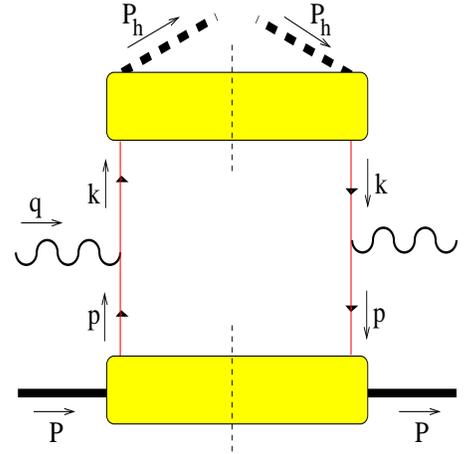
Assuming Factorization–Diagrammatic Approach

P.J. Mulders, R.D. Tangerman, NPB: 1996.

$$2M\mathcal{W}_{\mu\nu}(q; PS; P_h)$$

$$= \int d^4p d^4k \delta^4(p + q - k)$$

$$\text{Tr}(\Phi(p)\gamma_\mu\Delta(k)\gamma_\nu) + \left\{ \begin{array}{l} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{array} \right\}$$



• Distribution and Fragmentation Functions

$$\Phi(p, P) = \frac{1}{2} \sum_X \int \frac{d^3\xi}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}(\xi^-, \xi_\perp) \mathcal{G}_{[\xi^-, \infty]}^\dagger | X \rangle \cdot \langle X | \mathcal{G}_{[0, \infty]} \psi(0) | P \rangle |_{\xi^+ = 0}$$

$$\Delta(k, P_h) = \frac{1}{4z} \sum_X \int \frac{d^3\xi}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \mathcal{G}_{[\xi^+, -\infty]} \psi(\xi) | X; P_h \rangle \cdot \langle X; P_h | \bar{\psi}(0) \mathcal{G}_{[0, -\infty]}^\dagger | 0 \rangle |_{\xi^- = 0}$$

$$\Delta^{[i\sigma^{\perp-}\gamma_5]}(z, k_\perp) = \frac{1}{4z} \int dk^+ \text{Tr}(\gamma^- \gamma^\perp \gamma_5 \Delta) \Big|_{k^- = \frac{P_\perp}{z}}$$

$$\Phi^{[i\sigma^{\perp+}\gamma_5]}(x, p_\perp) = \frac{1}{2} \int dk^- \text{Tr}(\gamma^+ \gamma^\perp \gamma_5 \Phi) \Big|_{p^+ = xP}$$

T-Odd Contributions to Asymmetries

- T -odd quark distribution functions, e.g. $f_{1T}^\perp(x, k_\perp)$, $h_1^\perp(x, k_\perp)$, (Sivers PRD: 1990, Anselmino & Murgia PLB: 1995 ...) may exist as leading or higher twist effects due to initial/final state interactions or ...
- PRD: 1998 Boer and Mulders considered asymmetries, due to the presence of leading twist T -odd distribution functions, f_{1T}^\perp, h_1^\perp

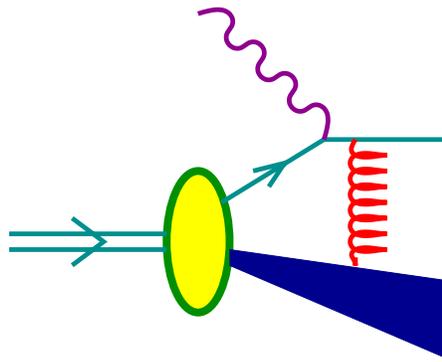
$$\begin{aligned}
 d\sigma_{\lambda,S} &\propto f_1 \otimes D_1 + \frac{k_T}{Q} f_1 \otimes D_1 \cdot \cos \phi \\
 &+ \left[\frac{k_T^2}{Q^2} f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp \right] \cdot \cos 2\phi \\
 &+ |S_T| \cdot h_1 \otimes H_1^\perp \cdot \sin(\phi + \phi_S) \quad \text{Collins} \\
 &+ |S_T| \cdot f_{1T}^\perp \otimes D_1 \cdot \sin(\phi - \phi_S) \quad \text{Sivers} \\
 &+ \dots
 \end{aligned}$$

$$\text{cos } 2\phi \text{ Asymmetry } A_{UU}^{\cos(2\phi)} \propto \frac{\sum_q e_q^2 h_1^{\perp(1)q}(x, Q^2) \cdot H_1^{\perp(1)q}(z, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) \cdot D_1^q(z, Q^2)}$$

$$\text{Sivers Asymmetry } A_{UT}^{\sin(\phi - \phi_S)} \propto \frac{\sum_q e_q^2 f_{1T}^{\perp(1)q}(x, Q^2) \cdot D_1^q(z, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) \cdot D_1^q(z, Q^2)}$$

Rescattering-Mechanism: T-Odd Contributions to Asymmetries

- **PLB: 2002** Brodsky, Hwang, and Schmidt demonstrate rescattering of a gluon could produce the necessary phase leading to nonzero SSAs at *Leading Twist*



- Ji and Yuan **PLB: 2002** describe effect in terms of gauge invariant distribution functions (**Collins, Soper NPB: 1982**)

$$\Rightarrow \langle P | \bar{\psi}(\xi^-, \xi_\perp) \mathcal{G}_{[\xi, \infty]}^\dagger | X \rangle \langle X | \mathcal{G}_{[0, \infty]} \psi(0) | P \rangle |_{\xi^+ = 0}$$

$$\mathcal{G}_{[\xi, \infty]} = \mathcal{G}_{[\xi_T, \infty]} \mathcal{G}_{[\xi^-, \infty]}, \quad \text{where} \quad \mathcal{G}_{[\xi^-, \infty]} = \mathcal{P} \exp\left(-ig \int_{\xi^-}^{\infty} d\xi^- A^+\right)$$

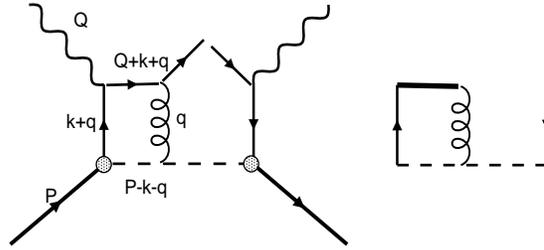
- Singular gauge, $A^+ = 0$, **effect remains**
Belitsky, Ji, Yuan NPB: 2003

- Collins, **PLB: 2002**, modifies earlier claim of trivial Sivers Effect
 $f_{1T}^\perp(x, k_\perp)|_{\text{DIS}} = -f_{1T}^\perp(x, k_\perp)|_{\text{DY}}$

Rescattering Mechanism to Generate T -Odd Function h_1^\perp

Goldstein, Gamberg–ICHEP-proc., Amsterdam: 2002, hep-ph/0209085

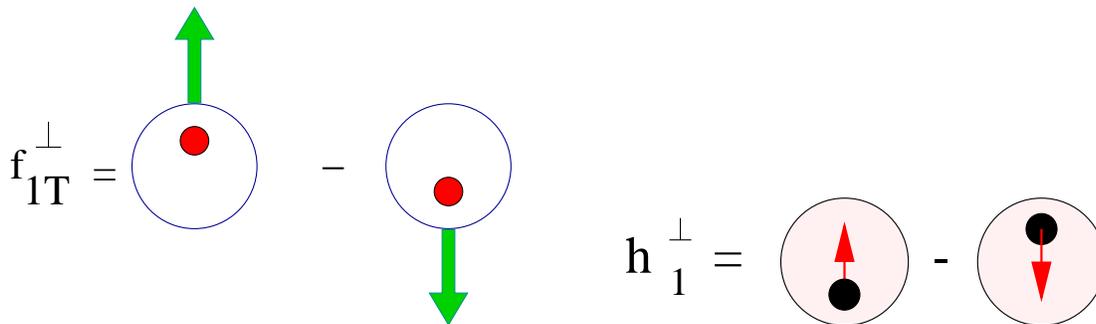
- h_1^\perp Naturally defined from gauge invariant TMPDF(s)
- Apply “eikonal Feynman rules”, (Collins, Soper, NPB: 1982)



$$\Phi^{[\Gamma]}(x, k_\perp) = \frac{1}{2} \sum_X \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp \vec{k}_\perp)} \langle P | \bar{\psi}(\xi^-, \xi_\perp) | X \rangle$$

$$\langle X | \left(-ie_1 \int_0^\infty A^+(\xi^-, 0) d\xi^- \right) \Gamma \psi(0, 0_\perp) | P \rangle \Big|_{\xi^+ = 0} + \text{h.c.}$$

- $h_1^\perp(x, k_\perp)$, represents, number density transversely polarized quarks in an unpolarized nucleons



- $f_{1T}^\perp(x, k_\perp)$, represents, number density *unpolarized* quarks in transversely polarized nucleons-complementary to h_1^\perp

Estimates of T-odd Contributions to Azimuthal Asymmetries

$$\cos 2\phi \text{ Asymmetry } A_{UU}^{\cos(2\phi)} \propto \frac{\sum_q e_q^2 h_1^{\perp(1)}(x, Q^2) \cdot H_1^{\perp(1)q}(z, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) \cdot D_1^q(z, Q^2)}$$

$$\text{Sivers Asymmetry } A_{UT}^{\sin(\phi - \phi_S)} \propto \frac{\sum_q e_q^2 f_{1T}^{\perp(1)q}(x, Q^2) \cdot D_1^q(z, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) \cdot D_1^q(z, Q^2)}$$

- ★ The quark-nucleon-spectator model used in previous rescattering calculations assumes point-like nucleon-quark-diquark vertex, leads to logarithmically divergent, asymmetries

Brodsky, Hwang, Schmidt, PLB: 2002;

Goldstein, L. Gamberg, ICHEP 2002;

Boer, Brodsky, Hwang, PRD: 2003

$$\begin{aligned} h_1^{\perp}(x, k_{\perp}) &= f_{1T}^{\perp}(x, k_{\perp}) \\ &= \frac{g^2 e_1 e_2 (1-x)(m+xM) M}{(2\pi)^4 4\Lambda(k_{\perp}^2)} \frac{M}{k_{\perp}^2} \ln \frac{\Lambda(k_{\perp}^2)}{\Lambda(0)} \end{aligned}$$

$$\Lambda(k_{\perp}^2) = k_{\perp}^2 + x(1-x) \left(-M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x} \right)$$

- Asymmetry involves weighted function

$$h_1^{\perp(1)}(x) \equiv \int d^2 k_{\perp} \frac{k_{\perp}^2}{2M^2} h_1^{\perp}(x, k_{\perp}^2) \quad \textit{diverges}$$

Log Divergence

Log divergence addressed by approximating the transverse momentum dependence of the quark-nucleon-vertex by a Gaussian distribution in k_{\perp}^2 ,

Gamberg, Goldstein, Oganessyan, PRD 67 (2003)

$$\langle n | \psi(0) | P \rangle = \left(\frac{i}{\not{k} - m} \right) \frac{b}{\pi} e^{-bk_{\perp}^2} U(P, S), \quad b \equiv \frac{1}{\langle k_{\perp}^2 \rangle}$$

$U(P, S)$ nucleon spinor, and quark propagator comes from untruncated quark line

Resulting in

$$h_1^{\perp}(x, k_{\perp}) = \frac{e_1 e_2 g^2 b^2 (m + xM)(1-x)}{2(2\pi)^4 \pi^2 \Lambda(k_{\perp}^2)} \frac{1}{k_{\perp}^2} \times e^{-b(k_{\perp}^2 - \Lambda(0))} \left[\Gamma(0, b\Lambda(0)) - \Gamma(0, b\Lambda(k_{\perp}^2)) \right]$$

$\Gamma(0, z) \equiv$ incomplete gamma function:

- Check approach: $\lim \langle k_{\perp}^2 \rangle \rightarrow \infty$ Gaussian width goes to infinity, regain *log divergent* result

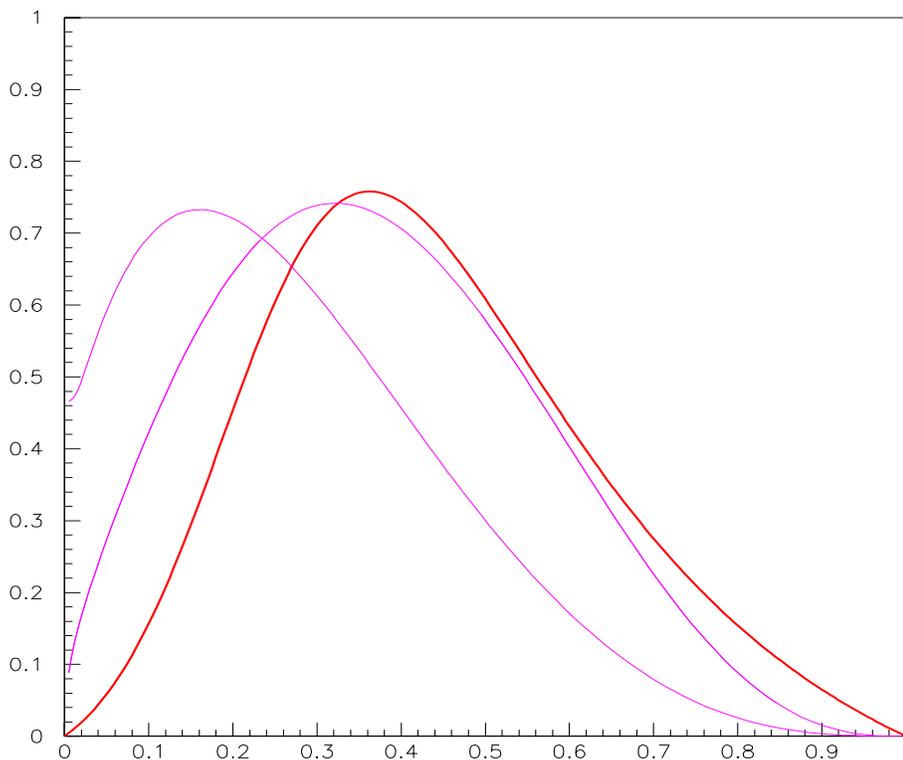
Unpolarized Structure Function

$$f(x) = \frac{g^2}{(2\pi)^2} \frac{b^2}{\pi^2} (1-x)$$

$$\cdot \left\{ \frac{(m+xM)^2 - \Lambda(0)}{\Lambda(0)} - \left[2b \left((m+xM)^2 - \Lambda(0) \right) - 1 \right] e^{2b\Lambda(0)} \Gamma(0, 2b\Lambda(0)) \right\}$$

★ Normalization, $\int_0^1 f(x) = 2$

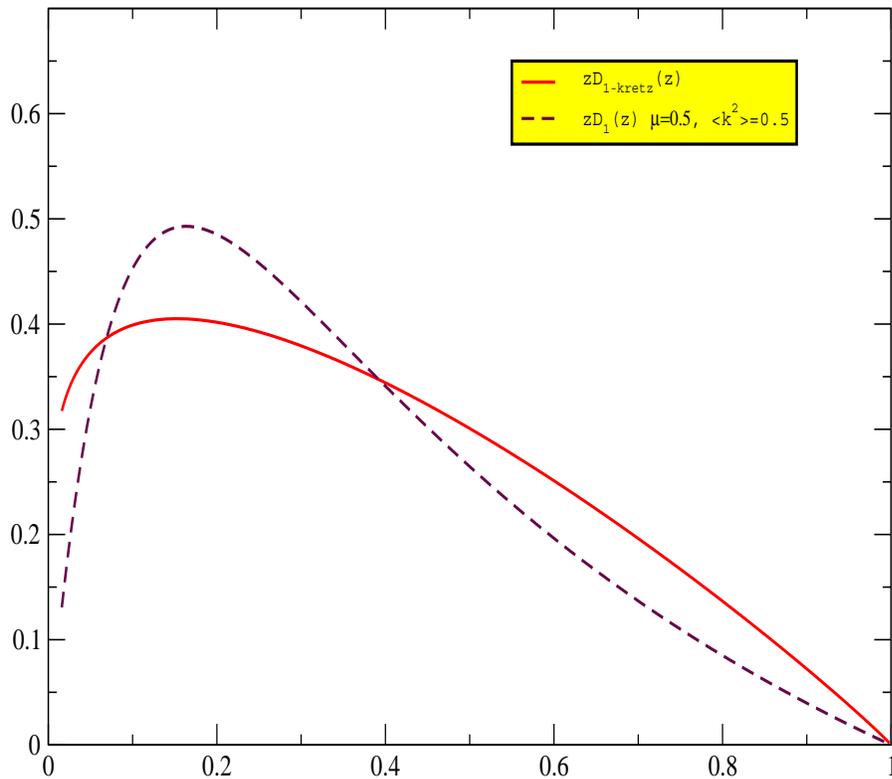
- thick full curve - xf_1
- thin full curve - xf_1 from GRV
- dot-dashed - xf_1 BBS



Pion Fragmentation Function

$$D_1(z) = \frac{N'^2 f_{qq\pi}^2}{4(2\pi)^2} \frac{1}{z} \frac{(1-z)}{z} \left\{ \frac{m^2 - \Lambda'(0)}{\Lambda'(0)} - \left[2b' (m^2 - \Lambda'(0)) - 1 \right] e^{2b'\Lambda'(0)} \Gamma(0, 2b'\Lambda'(0)) \right\},$$

which, multiplied by z at $\langle k_{\perp}^2 \rangle = (0.5)^2 \text{ GeV}^2$ and $\mu = m$, is in good agreement with the distribution of [Kretzer, PRD: 2000](#)



Double T-odd $\cos 2\phi$ asymmetry

Transversity of quarks inside an unpolarized hadron, and $\cos 2\phi$ asymmetries in unpolarized semi-inclusive DIS

Gamberg, Goldstein, Oganessian PRD 2003

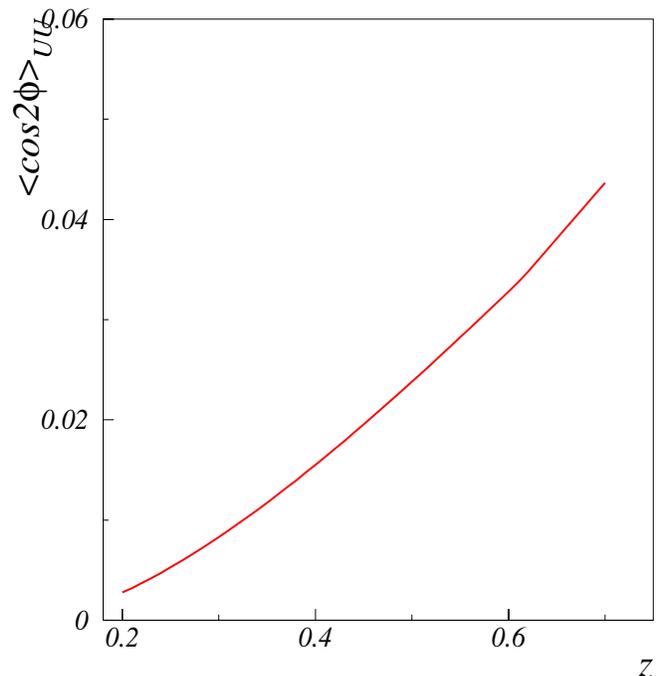
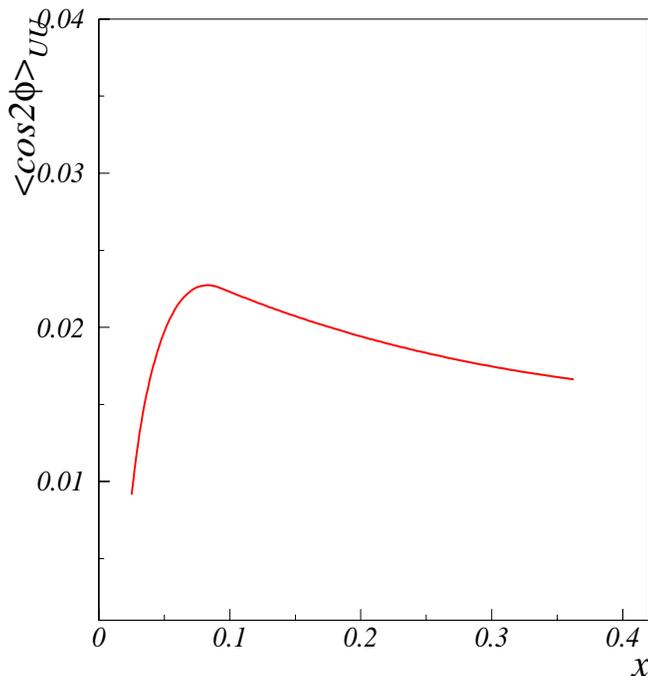
For the HERMES kinematics

$$1 \text{ GeV}^2 \leq Q^2 \leq 15 \text{ GeV}^2, 4.5 \text{ GeV} \leq E_\pi \leq 13.5 \text{ GeV},$$

$$0.2 \leq z \leq 0.7, 0.2 \leq y \leq 0.8, \langle P_{h\perp}^2 \rangle = 0.25 \text{ GeV}^2 \text{ and } \langle P_{h\perp} \rangle = 0.4 \text{ GeV}$$

$$\begin{aligned} \left\langle \frac{|P_{h\perp}^2|}{MM_h} \cos 2\phi \right\rangle_{UU} &= \frac{\int d^2 P_{h\perp} \frac{|P_{h\perp}^2|}{MM_h} \cos 2\phi d\sigma}{\int d^2 P_{h\perp} d\sigma} \\ &= \frac{8(1-y) \sum_q e_q^2 h_1^{\perp(1)}(x) z^2 H_1^{\perp(1)}(z)}{(1+(1-y)^2) \sum_q e_q^2 f_1(x) D_1(z)} \end{aligned}$$

Used Collins Ansatz, Collins, NPB: 1993; Kotzinian, Mulders, PLB: 1997 and GRV Phys. Rep. 1981 for $D_1(z)$ parameterization

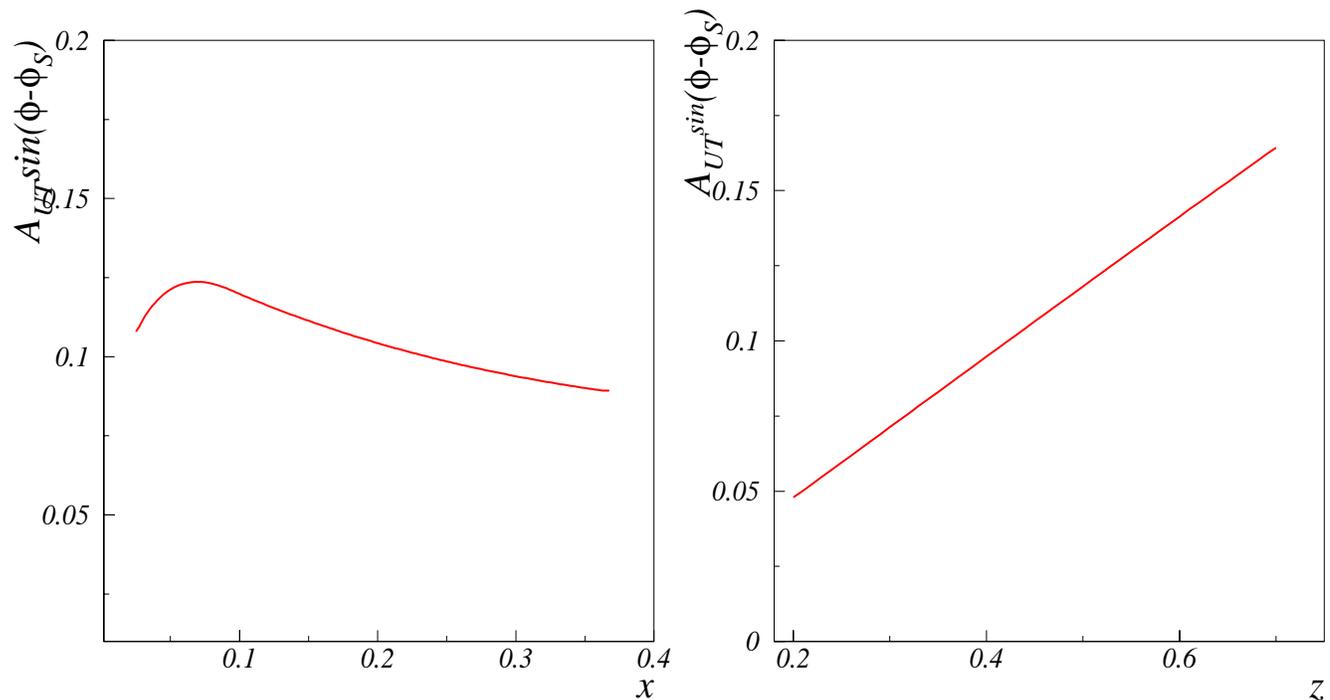


Estimates for Sivers Asymmetry

$$\begin{aligned} \left\langle \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S) \right\rangle_{UT} &= \frac{\int d^2 P_{h\perp} \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S) d\sigma}{\int d^2 P_{h\perp} d\sigma} \\ &= \frac{(1 + (1 - y)^2) \sum_q e_q^2 f_{1T}^{\perp(1)}(x) z D_1^q(z)}{(1 + (1 - y)^2) \sum_q e_q^2 f_1(x) D_1(z)}, \end{aligned}$$

where

$$A_{UT}^{\sin(\phi - \phi_S)} \approx \frac{M}{\langle P_{h\perp} \rangle} \left\langle \frac{|P_{h\perp}|}{M} \sin(\phi - \phi_S) \right\rangle_{UT}$$



Rescattering Mechanism for T-Odd Collins Function

Gamberg, Goldstein, Oganessyan hep-ph/0307139, PRD68, 2003

- Gauge-link contribution to the Collins Function:

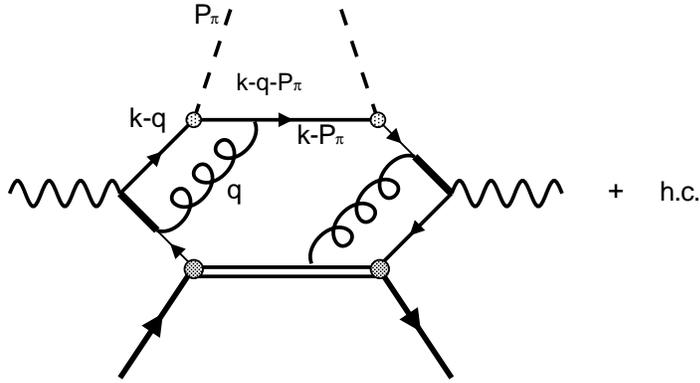


Figure depicts $h_1^\perp \star H_1^\perp \cos 2\phi$ asymmetry. The momenta flow to the quark-pion vertex is shown. The momentum q is the loop integration variable.

We evaluate the projection $\Delta^{[i\sigma^\perp - \gamma_5]}$, which results in the leading twist, contribution to T -odd pion fragmentation

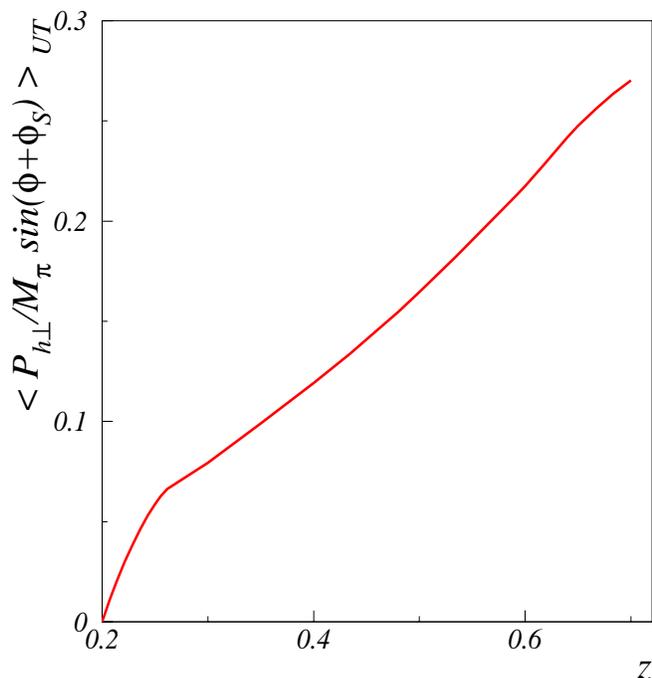
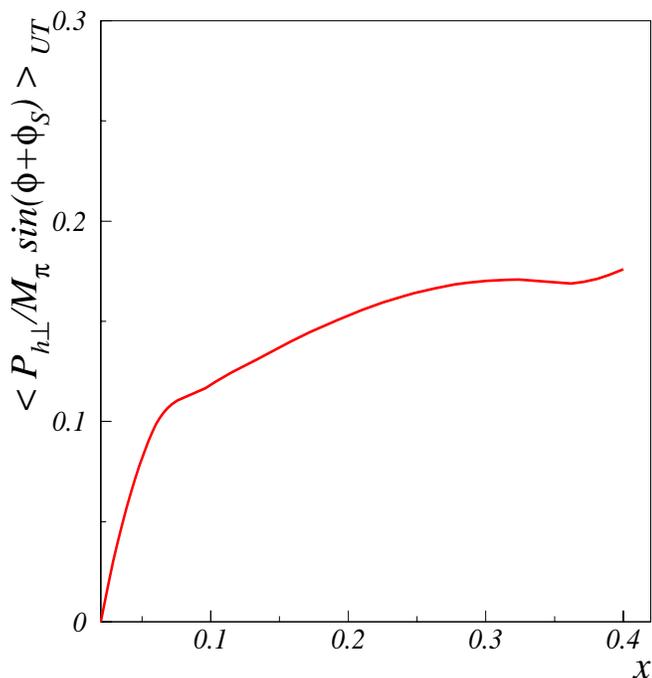
$$H_1^\perp(z, k_\perp) = \frac{N'^2 f^2 g^2}{(2\pi)^4} \frac{1}{4z} \frac{(1-z)}{z} \frac{m}{\Lambda'(k_\perp^2)} \frac{M_\pi}{k_\perp^2} e^{-b'(k_\perp^2 - \Lambda'(0))} \left[\Gamma(0, b\Lambda'(0)) - \Gamma(0, b'\Lambda'(k_\perp^2)) \right],$$

$$\text{where, } \Lambda'(k_\perp^2) = k_\perp^2 + \frac{1-z}{z^2} M_\pi^2 + \frac{\mu^2}{z} - \frac{1-z}{z} m^2$$

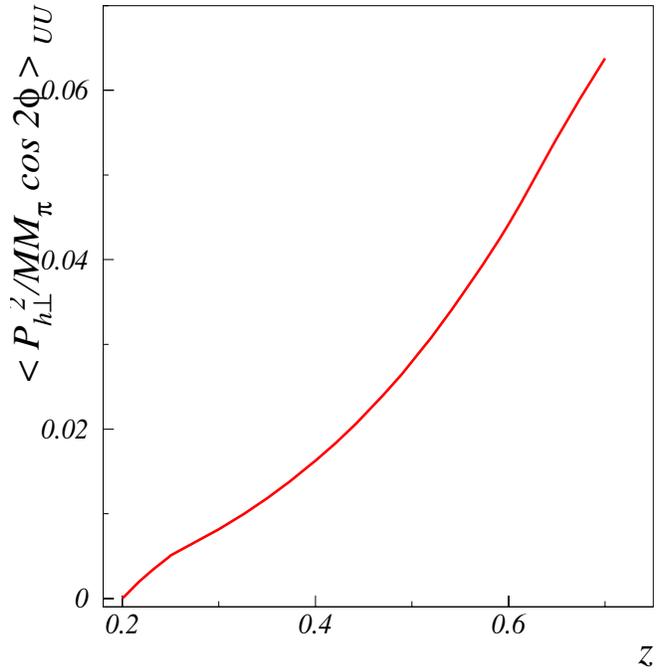
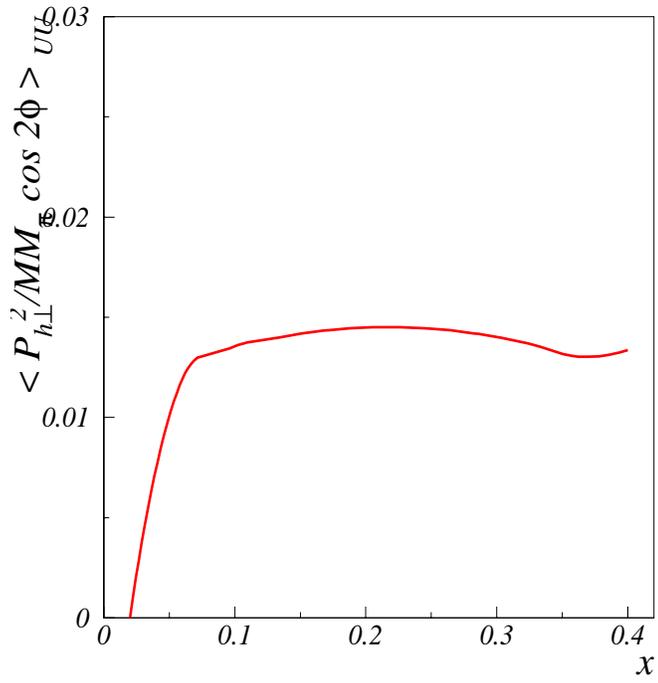
Collins Asymmetry

- Convolution of two chiral-odd (both T -odd and T -even) structures,

$$\begin{aligned} \left\langle \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_s) \right\rangle_{UT} &= \frac{\int d\phi_s \int d^2 P_{h\perp} \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_s) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_s \int d^2 P_{h\perp} (d\sigma^\uparrow + d\sigma^\downarrow)} \\ &= |S_T| \frac{2(1-y) \sum_q e_q^2 h_1(x) z H_1^{\perp(1)}(z)}{(1 + (1-y)^2) \sum_q e_q^2 f_1(x) D_1(z)}. \end{aligned}$$



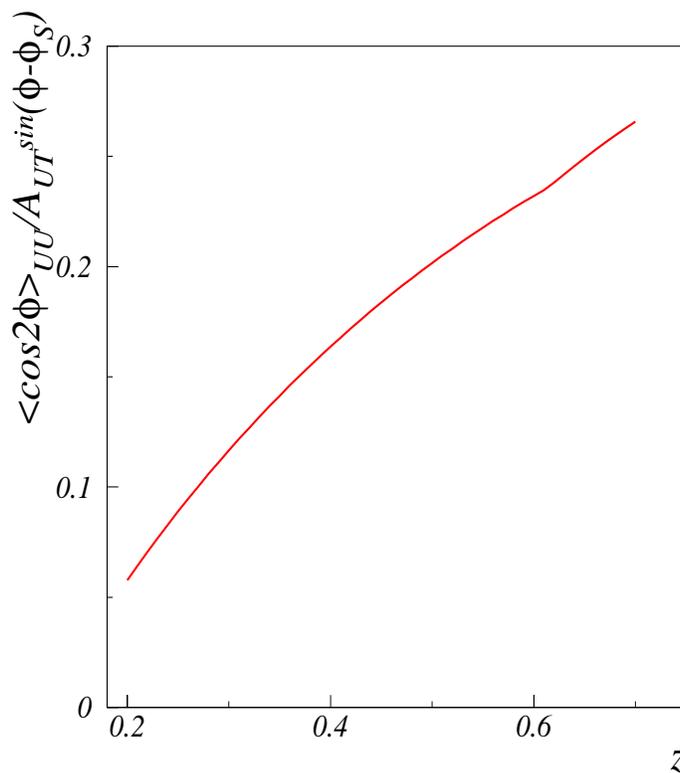
$\cos 2\phi$...



The T -odd $\cos 2\phi$ vs Siverson SSA

$$A = \frac{\langle \cos 2\phi \rangle_{UU}}{A_{UT}^{\sin(\phi - \phi_S)}} \sim z \cdot \frac{H_1^{\perp(1)}(z)}{D_1(z)},$$

Reflected in equality of h_1^{\perp} and f_{1T}^{\perp} functions in diquark model. Supports suggestion that the single-spin $\sin(\phi - \phi_S)$ Siverson and spin-independent $\cos 2\phi$ asymmetries are closely related in hard scattering processes

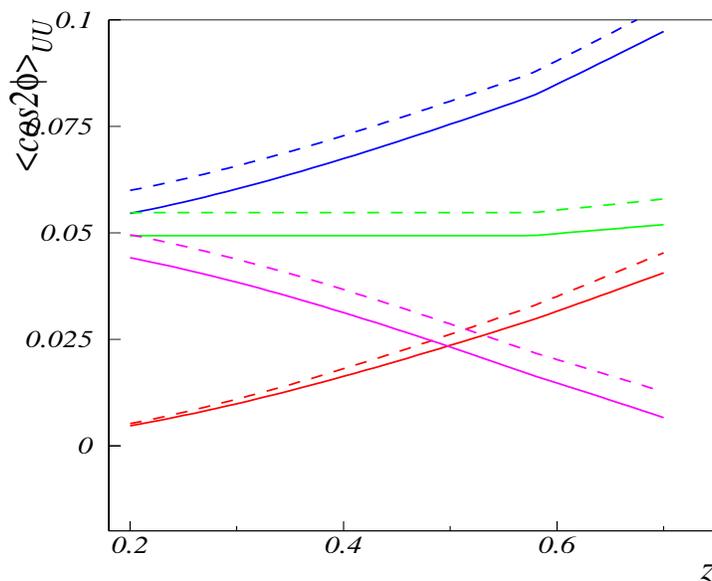


Combined $\cos 2\phi$ asymmetry

Effects that vanish as M^2/Q^2 important at small and moderate values of $Q^2 \Rightarrow \langle \cos 2\phi \rangle_{UU}$ results from the ordinary kinematic twist-2 T -even and leading double T -odd effects,

Gamberg, Goldstein, Oganessyan:DIS-2003-hep/ph-arXiv

$$\langle \cos 2\phi \rangle_{UU} = \frac{2 \frac{\langle k_{\perp}^2 \rangle}{Q^2} f_1(x) D_1(z) \pm 8(1-y) h_1^{\perp(1)}(x) H_1^{\perp(1)}(z)}{\left[1 + (1-y)^2 + 2 \frac{\langle k_{\perp}^2 \rangle}{Q^2} (1-y) \right] f_1(x) D_1(z)}$$



The z -dependences of the asymmetry The full and dashed curves correspond to the asymmetry with and without k_{\perp}^2/Q^2 term in the denominator, respectively.

Red \Rightarrow T -odd term.

Green \Rightarrow T -even term

Blue \Rightarrow sum of them

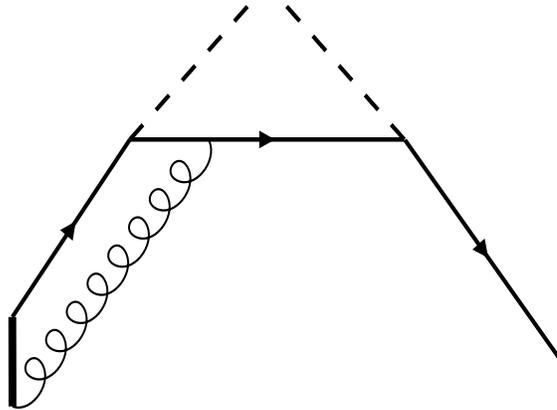
Magenta \Rightarrow difference

DRELL YAN

- See Boer Brodsky Hwang PRD:2003
- Gamberg Goldstein Oganessyan: Higher Twist: In prep

In this approach other contribution ...

On the Subject of other contributions in this approach
see [Bacchetta, Metz, Yang PLB2004](#)



- Can mass corrections generate necessary phases?
- Must be careful about scales off shellness goes to on shell in deriving mass singularities or collinear singularities.
- We are finally confronting non-perturbative physics with PQCD....

$$\log \frac{Q^2}{\mu^2}$$

See new work of Ji, Ma, Yuan hep-ph/0404183

SUMMARY

- The angular correlations in semi-inclusive DIS are considered as a source of “rescattering” to generate T-odd, intrinsic transverse momentum, k_{\perp} , dependent *distribution and fragmentation* functions at leading twist
- We have evaluated these functions by modeling the quark, spectator hadron vertices in a quark-diquark-hadron framework
- We have evaluated azimuthal and SSA with Gaussian “regularization” in $\langle k_{\perp} \rangle$ and addressed the **Log divergence problem**
- In particular, analyzing the leading twist contribution to the $\cos 2\phi$ azimuthal asymmetries we have considered the impact that novel T-odd distribution and fragmentation functions have on transversity “within” nucleon
- We consider the implications that these T-odd distribution and fragmentation functions have in Sivers and Collins asymmetries
- ★ Azimuthal asymmetries and SSA measured at HERMES and SMC and possibly JLAB hopefully will reveal the extent to which these effects impact the data
- These experiments may point to the essential role played by quark transverse momentum and T -odd quark distributions and effects of higher twist